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Comprehensive evaluation of performance of ARALL by fuzzy multi-aspect decision making method

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ARALL (Aramid Aluminium Laminate) is a super-hybrid composite, and there are many factors influencing the performances of the laminate. In this paper, fuzzy theory is applied to deal with this composite. Based on experimental results, the factors which influence the properties of ARALL, including the residual stresses, adhesive properties and adhesive content, are analysed by fuzzy multi-criteria. The results show that the state of the residual stress is the key factor which influences the performance of ARALL. The adhesive properties followed by the adhesive content in the composite also play a role in determining the properties of the composite.

1. Introduction

ARALL is a super-hybrid composite which has great potential for use in aircrafts [1–3]. There are many factors which influence the performance of ARALL [4–7]. In reference [8], the effects on interlaminar performances, tensile properties of the laminate with notch, fatigue crack growth rate and visco-elastic behaviour, by its residual stresses and adhesive were analysed. The evaluation of these factors and their effects is an area for analysis with multi-attribute decision making, since generally this kind of problem is indistinct, or fuzzy. The adhesive content, for example, has some effect on the interlaminar performance of ARALL, and has a smaller effect on fatigue crack growth rate, but has strong effects on the notch sensitivity of ARALL. Obviously, there are no exact determinations in this result, and since the factors are inter-related there is no strict boundary between them. In other words, when many properties are taken into account simultaneously, it is difficult to determine the order in which the factors influence the comprehensive properties of ARALL. So, to analyse these problems, fuzzy theory is applied [9].

2. Figure of factors' relation and fuzzy matrix

The relation of the factors can be shown by Fig. 1. In order to make the figure clear, only two factors (σ_{res} the residual stress; and V_m the adhesive content) are considered. The interlaminar properties, notch sensitivity index, fatigue crack growth rate, dynamic mechanical property and creep are defined as G, NSI, da/dN, $tg\delta$ and ε_c respectively, where, set $X = \{\sigma_{res}, V_m\}$ and set $Y = \{\sigma_{eff}, W_{int}\}$.

σ_{eff} stands for the effective stress of ARALL under loading; W_{int} stands for the failure work of fibre resin layer. Set $Z = \{G, NSI, da/dN, tg\delta, \varepsilon_c\}$

Fig. 1 shows that the residual stress and adhesive content have effects on the above properties through σ_{eff} and W_{int}. If more factors are taken into account, the figure will become very complicated. So instead a matrix representation will be used.

Suppose

$$X = \{x_1, x_2, x_3, x_4, x_5\} \quad (1)$$

where, x_1: residual stress in Al layer of ARALL
x_2: residual stress in aramid/adhesive layer
x_3: toughness of adhesive
x_4: adhesion between adhesive and fibre
x_5: adhesive contents.

It is considered that the factors in set X have effects on set Z through σ_{eff} and W_{int}. Suppose:

$$Y = \{y_1, y_2\} \quad (y_1 - \sigma_{eff}, y_2 - W_{int}) \quad (2)$$

$$Z = \{z_1, z_2, z_3, z_4, z_5\}$$

$$(z_1 - G, z_2 - NSI, z_3 - da/dN, z_4 - tg\delta, z_5 - \varepsilon_c) \quad (3)$$

The relation of X and Y:

$$P = X \times Y = \{\langle x_1, y_1\rangle \langle x_2, y_1\rangle \langle x_3, y_2\rangle \times \langle x_4, y_2\rangle \langle x_5, y_2\rangle\}$$

The relation of Y and Z:

$$Q = Y \times Z = \{\langle y_1, z_1\rangle \langle y_1, z_2\rangle \langle y_1, z_3\rangle \times \langle y_1, z_4\rangle \langle y_1, z_5\rangle \langle y_2, z_1\rangle \times \langle y_2, z_2\rangle \langle y_2, z_3\rangle \langle y_3, z_4\rangle\}$$

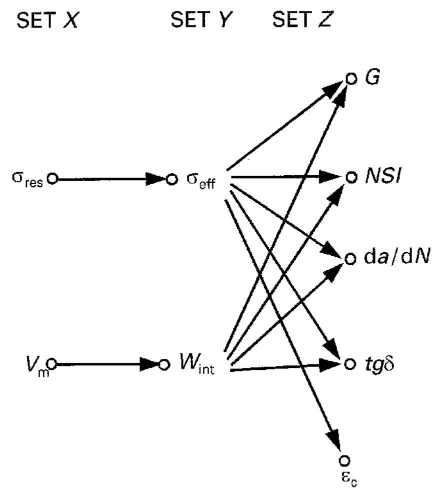

Figure 1 Multi-factor relation.

P and Q are expressed as a Boolean matrix:

$$P(p_{ij}) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \end{array}$$

$$Q(q_{ij}) = \begin{array}{c} \\ y_1 \\ y_2 \end{array} \begin{array}{c} \begin{array}{ccccc} z_1, & z_2, & z_3, & z_4, & z_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Through the operations of $R(r_{i,j}) = P \cdot Q$, the relation of X and Z can be derived:

$$r_{ij} = \bigvee_{k=1}^{5} (p_{ik} \wedge q_{kj}) \quad i = 1, 2, \ldots, 5$$

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{array}{ccccc} z_1, & z_2, & z_3, & z_4, & z_5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

From matrix R, it can be determined that four kinds of properties apart from creep performances are influenced by residual stress, adhesive properties and adhesive content, and the creep of ARALL is mainly influenced by the state of residual stress.

In order to obtain a fuzzy matrix for use in multi-attribute making, it should turn the elements of the matrices P and Q into values between [0, 1]. So a gradation method is applied. That is, the effect coefficient which a component in a set (X or Y) has on a component in another set (Y or Z) can be:

biggest	1
bigger	0.75
smaller	0.50
small	0.25
none	0

Following the results of reference [9] and the above gradation, the results are normalized, and the fuzzy relation of matrix $\tilde{P}$ and $\tilde{Q}$ is deduced as:

$$\tilde{P} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \\ 0 & 0.4 \\ 0 & 0.5 \\ 0 & 0.1 \end{bmatrix}$$

$$\tilde{Q} = \begin{bmatrix} 0.2 & 0.6 & 0.5 & 0.4 & 0.9 \\ 0.8 & 0.4 & 0.5 & 0.6 & 0.1 \end{bmatrix}$$

Through the operation of union of fuzzy matrix:

$$\tilde{R} = \tilde{P} \cdot \tilde{Q} = \begin{bmatrix} 0.2 & 0.5 & 0.5 & 0.4 & 0.5 \\ 0.2 & 0.5 & 0.5 & 0.4 & 0.5 \\ 0.4 & 0.4 & 0.4 & 0.4 & 0.1 \\ 0.5 & 0.4 & 0.5 & 0.5 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix} \quad (4)$$

3. Multi-attribute decision model

Suppose that the factor set is

$$Z = \{z_1, z_2, \ldots, z_m\}$$

and the comment set is

$$V = \{v_1, v_2, \ldots, v_n\}$$

The fuzzy relation between Z and V can be expressed as the evaluation matrix:

$$\tilde{R} = \begin{bmatrix} R_1 \\ R_2 \\ \\ R_m \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix} \quad (5)$$

If a fuzzy vector and fuzzy matrix R are given, applying the operation of union on them, leads to:

$$\tilde{A} \cdot \tilde{R} = \tilde{B} \quad (6)$$

or

$$(a_1 \quad a_2 \quad \ldots \quad a_m) \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}$$

$$= (b_1 \quad b_2 \quad \ldots \quad b_n) \quad (7)$$

where

$$\tilde{B} = (b_1 \quad b_2 \quad \ldots \quad b_n) \quad (8)$$

This is the multi-criteria analysis result.

Due to the many factors to be considered and the point that these factors belong to a different arrangement: x_1 and x_2 belong to the state of stress; x_3 to fracture property of component materials; a two-step multi-attribute decision model is applied, and the extra information brought by each factor can be absorbed comprehensively.

The steps are:

(i) Divide the factor set Z into S subsets (unrelated) according to their natures.

$$Z = \{z_1, z_2, \ldots, z_s\}$$

(ii) According to the elements function in $Z_k = (z_{k1}, z_{k2}, \ldots, z_{km}\}$ $(k = 1, 2, \ldots, s)$, fuzzy set $A_k = (a_{k1}, a_{k2}, \ldots, a_{km})$ is defined, then by operation of union:

$$A_k R_k = B_k = (b_{k1}, b_{k2}, \ldots, b_{km}) \quad (k = 1, 2, \ldots, s) \tag{9}$$

This is the first step decision.

(iii) Take S subsets $Z_k (k = 1, 2, \ldots, S)$ as S factors of Z, and according to the function of Z_k in Z fuzzy vector is formed:

$$A = (a_1, a_2, \ldots, a_s) \tag{10}$$

And in Equation 9

$$B_k = (b_{k1}, b_{k2}, \ldots, b_{km}) \quad (k = 1, 2, \ldots, s)$$

is defined as the general decision matrix:

$$\tilde{R} = \begin{bmatrix} B_1 \\ B_2 \\ \\ B_s \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ b_{m1} & b_{m2} & \cdots & b_{mn} \end{bmatrix} \tag{11}$$

then leads to

$$\tilde{B} = \tilde{A} \cdot \tilde{R} = \tilde{A} \cdot \begin{bmatrix} B_1 \\ B_2 \\ \\ B_s \end{bmatrix} = A \begin{bmatrix} A_1 & R_1 \\ A_2 & R_2 \\ \\ A_s & R_s \end{bmatrix} \tag{12}$$

This is the two-stage multi-aspect decision model, where:

$$\tilde{B} = (b_1, b_2, \ldots, b_n)$$

is the final decision result.

4. Comprehensive evaluation

The decision object set is:

$$X = \{x_1, x_2, x_3, x_4, x_5\}$$

and the decision aspect set is:

$$Z = \{z_1, z_2, z_3, z_4, z_5\} \quad z_i \ (i = 1, \ldots, 5)$$

is the decision standard.

Through the matrix R (Equation 4):

$$\tilde{R} = \begin{bmatrix} 0.2 & 0.5 & 0.5 & 0.4 & 0.5 \\ 0.2 & 0.5 & 0.5 & 0.4 & 0.5 \\ 0.4 & 0.4 & 0.4 & 0.4 & 0.1 \\ 0.5 & 0.4 & 0.5 & 0.5 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix}$$

the decision space $S = (X, Z, R)$ is obtained. And the fuzzy set is supposed to consist of non-dominated alternatives, or $a_1 = a_2 = \cdots = a_5 = \frac{1}{5}$; by means of the operation of union of fuzzy sets, and applying $M(\cdot, +)$ model, the decision index is:

$$\tilde{M}_e = \tilde{R} \cdot \tilde{A}$$

$$= \begin{bmatrix} 0.2 & 0.5 & 0.5 & 0.4 & 0.5 \\ 0.2 & 0.5 & 0.5 & 0.4 & 0.5 \\ 0.4 & 0.4 & 0.4 & 0.4 & 0.1 \\ 0.5 & 0.4 & 0.5 & 0.5 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix} \begin{bmatrix} 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \end{bmatrix} = \begin{bmatrix} 0.42 \\ 0.42 \\ 0.36 \\ 0.38 \\ 0.1 \end{bmatrix}$$

Then maximizing and minimizing R leads to:

$$M_{max} = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \\ 0.5 \\ 0.1 \end{bmatrix} \quad M_{min} = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

order: $U_1 = (M_e, M_{max}, M_{min})$

and derive new decision space: $S_1 = (X_1, U_1, R_1)$, where R_1 is:

R_1	M_e	M_{max}	M_{min}
x_1	0.42	0.5	0.2
x_2	0.42	0.5	0.2
x_3	0.36	0.5	0.1
x_4	0.38	0.5	0.1
x_5	0.1	0.1	0.1

we then make the second decision.

Suppose there is no obvious difference amongst M_e, M_{max} and M_{min}, and order $a_1 = a_2 = a_3 = \frac{1}{3}$, and the operation model still has the form of: $M(\cdot, +)$, then:

$$M = \begin{bmatrix} 0.42 & 0.5 & 0.2 \\ 0.42 & 0.5 & 0.2 \\ 0.36 & 0.5 & 0.1 \\ 0.38 & 0.5 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix} \cdot \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 0.37 \\ 0.37 \\ 0.32 \\ 0.33 \\ 0.1 \end{bmatrix}$$

Therefore, the order (from strong to weak) which the decision object term influences the laminate is:

$$x_1 = x_2 > x_4 > x_3 > x_5$$

or, the heaviest is the residual stress. So, in order to obtain a laminate with good comprehensive properties, the residual stresses are first taken into account. Secondly, about choosing adhesive, the adhesion ability of the adhesive with the fibre is important, then the toughness of the adhesive; and finally, the adhesive content should be properly chosen.

5. Conclusion

Fuzzy theory is applied in this paper to a composite laminate. The effects on the properties of ARALL by residual stress and adhesive are expressed by a factor's relation figure and a fuzzy decision matrix. Through the operation of a fuzzy set, the decision matrix is derived. By applying the two-step multi-attribute decision model, the order of effects that the residual stresses, adhesive properties and its content have on the performances of ARALL is analysed. The result shows that the residual stress is the key factor influencing the performance of ARALL, followed by the adhesive's mechanical-energy property, the adhesive toughness and adhesive content.

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